$\hat{Q}_i$ (i = 1, 2, 3, 4) is the annihilation operator of the i-th Q-spurion (or the creation operator of an anti-Q-spurion), and $\hat{Q}_i^\dagger$ (i = 1, 2, 3, 4) is the creation operator of the i-th Q-spurion (or the annihilation operator of an anti-Q-spurion). The Q-spurions are auxiliary fictitious particles having the following properties:

1) The operators $\hat{Q}_i$ and $\hat{Q}_i^\dagger$ commute with the field operators.

2) If Ψ_1 and Ψ_2 are vectors in Hilbert state space, then

$$(\Psi_1, \hat{Q}_i^\kappa \Psi_2) = (\Psi_1, \hat{Q}_i^{+\kappa} \Psi_2) = (\Psi_1, \Psi_2),$$

where κ is a positive integer (or zero).

3) Under the transformations (9a)-(11a) the operators Q_i are transformed according to the laws

$$\hat{Q}_i \to \hat{Q}'_i = e^{(i/2)\gamma\beta}\hat{Q}_i; \quad \hat{Q} \to \hat{Q}' = e^{(i/2)\gamma\beta}\hat{Q}; \tag{9d}$$

$$\hat{Q}_i \to \hat{Q}'_i = \hat{Q}_i; \quad \hat{Q} \to \hat{Q}' = \exp\left\{\frac{i}{2} g \hat{\tau}_\kappa \omega_\kappa\right\}\hat{Q}; \tag{10c}$$

$$\hat{Q}_i \to \hat{Q}'_i = \hat{Q}_i; \quad \hat{Q} \to \hat{Q}' = \exp\left(\frac{iG}{2}\hat{T}_\kappa \Omega_\kappa\right)\hat{Q}. \tag{11b}$$

The Lagrangian for the entire system of fields has the form

$$L = L(q, n, B_\mu, \mathbf{W}_\mu, \mathbf{S}_\mu) + L(\varphi) + L(n, q, \varphi). \tag{24}$$

In our next article we shall investigate the vacuum corresponding to this Lagrangian.

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CHANGE IN THE TOPOLOGY OF PHYSICAL SPACE IN A CLOSED UNIVERSE

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The conditions under which physical space alters the number of connected components are determined.

The classical representation of physical space assigns it a connectedness, which is a fundamental topological property. Physical space, which is in essence a three-dimensional connected manifold, is combined with the time to form a common four-dimensional space–time. If we now consider a model of a connected, but not singly connected, space–time, then it is quite possible that we may observe some unconnected three-dimensional spacelike cross sections. Furthermore, an unconnected cross section M_1 can be obtained from a connected one M_0 through a spherical change in structure [1], so that a connected cross section and an unconnected one may be thought of as the initial and final states of some geometrodynamic process (a Lorentzian cobordism [1]). In the course of this process, the three-geometry undergoes a transition through a critical state $M_{1/2}$, which corresponds to a disruption of the connectedness of the spacelike cross section.

It would be interesting [1] to determine under which conditions the connectedness of the spacelike cross sections is disrupted; or, if we put aside the question of the particular differential-topology model, it would be interesting to determine whether the three-dimensional

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space M_0 becomes unconnected in the course of some physical process. Loosely speaking, we could say that a disruption of the connectedness means that a region D_0 is torn away from M_0.

The transformation from M_0 to M_1 can be performed by contracting to a point α^* the boundary ∂D_0 of the closed region $D_0 \subset M_0$. The result is the space $M_{1/2} = C_{1/2} \bigcup D_{1/2}$, where $C_{1/2}$ and $D_{1/2}$ have a single common point, α^* (the result of the contraction of ∂D_0) and are connected components of the space M_1. At this point, $D_{1/2}$ is torn away from $C_{1/2}$, and we find M_1.

Geometrically, the disruption of connectedness may be characterized as a decrease to zero of the area of the surface ∂D_0 which bounds the region which is torn away, D_0. This means that the connectedness of the space is disrupted by a perturbation of the metric, $\gamma_{\alpha\beta} \to \gamma_{\alpha\beta} + \delta\gamma_{\alpha\beta}$ (α, β = 1, 2, 3). A local perturbation of the metric leads to a change in the curvature of three-space. In the general theory of relativity, three-space is treated as a spacelike cross section of space–time. We should therefore work from a perturbation of the four-metric g_{ik} (i, k = 0, 1, 2, 3) of the space–time which initiates a perturbation of the metric $\gamma_{\alpha\beta}$ of three-space. According to Einstein's equations, the initial cause of the perturbation of the metric is the appearance of an additional local energy source. The energy expenditure which would be necessary to disrupt the connectedness of three-space could easily be calculated if we had an equation relating some numerical characteristic of the connectedness of a space to the curvature of this space.

In the case of a closed three-space M, a numerical characteristic meeting this description is the zero-dimensional Betti number β_0 (M) [2]. We also have the necessary equation, but admittedly only for the particular case of a closed, oriented, Riemannian three-space M with the metric $\gamma_{\alpha\beta}$ which permits a regular unique Killing vector field ξ [3]:

$$\frac{1}{2\pi l(\xi)} \int\limits_{M} \{K(\xi^{\perp}) - 3K(\xi)\}\, dv = 2\beta_0(M) - \beta_1(M) + d_0, \tag{1}$$

where d_0 = 0 or 1, depending on whether the one-dimensional Betti number $\beta_1(M)$; $K(\xi^{\perp})$ is of even or odd parity; ; K (ξ) is the Riemannian curvature in the plane orthogonal to ξ; $K(\xi)$ is the Riemannian curvature for an arbitrary plane which contains ξ [we note that $K(\xi)$ does not depend on the choice of this plane]; dv is the volume form; and $l(\xi)$ is the length of the integral path of the field ξ (a constant).

We tear the region D_0 away in the following manner. On the three-manifold M_0 we specify a family of Riemannian metrics $\gamma_{\alpha\beta}(t)$, $t \in [0, 1]$, which satisfies the following conditions.

a) For $0 \leqslant t < 1/2$, $\gamma_{\alpha\beta}(t)$ is a C^2-smooth tensor field, while at $t \geqslant 1/2$ it has discontinuities in its first order derivatives at the boundary ∂D_0 of the closed region D_0;

b) (Contraction of ∂D_0 to a point α^*). The area σ_t of the boundary ∂D_0 calculated in the metric $\gamma_{\alpha\beta}$ (t) tends toward zero in the limit $t \to 1/2 - 0$; in other words,

$$dv_t|_{\partial D_0} \underset{t\to 1/2-0}{\longrightarrow} 0 \text{ and } dv_t|_{\partial D_0} = 0 \quad \text{for } t \geqslant 1/2,$$

where d_{vt} is the volume form in the metric $\gamma_{\alpha\beta}(t)$; $dv_s/dv_t \leqslant 1$ on M_0, $t < \frac{1}{2} < s$;

c) The space $< M_0, \gamma_{\alpha\beta}(0) >$, i.e., M_0 with the metric $\gamma_{\alpha\beta}$ (0), is a connected C^2-smooth Riemannian manifold, while $C_t \equiv (M_0 \setminus D_0) \cup \{\alpha^*\}$ and $D_t \equiv D_0 \cup \{\alpha^*\}$ with the metric $\gamma_{\alpha\beta}(t)$, $t \geqslant 1/2$, and supplemented with the point α^* are C^2-smooth connected Riemannian closed manifolds.

d) The $\partial\gamma_{\alpha\beta}/\partial n$, where n is the normal to the space $< M_0, \gamma_{\alpha\beta}(t) >$, are continuous.

e) We have $\gamma_{\alpha\beta}(t) = \gamma_{\alpha\beta}(0)$ outside the neighborhood O_ε of the region D_0;

f) The space $< M_0, \gamma_{\alpha\beta}(t) >$, $t > 1/2$ has a nonnegative curvature.

g) The space $< M_0, \gamma_{\alpha\beta}(t) >$, $t \in [0, 1]$ permits a regular unique Killing field ξ_t.

The last of these assumptions is the least attractive, since as D_0 is being torn away from M_0 the symmetry of the three-space may apparently disappear as the critical value t = 1/2 is approached. However, understanding this point, we are forced to introduce condition g, so that we may use Eq. (1). Yodzis [1] has pointed out that it is necessary to assume a symmetry as a means for making some sort of progress toward a solution of our problem.

We will use the index "t" to indicate entities which correspond to the space $< M_0, \gamma_{\alpha\beta}(t) >$.

For simplicity we assume that we always have $< M_0, \gamma_{\alpha\beta}(t) >$ The space t < 1/2 is connected, so that ξ_1 0.

$$\int_M f(\xi_t)\,dv_t = 4\pi l(\xi_t), \tag{2}$$

where

$$f(\xi_t) = K(\xi_t^{\perp}) + 3K(\xi_t).$$

At s > 1/2 the space $< M_0, \gamma_{\alpha\beta}(s) >$ has two connected components. Consequently,

$$\int_{D_s} f(\xi_s)\,dv_s = 4\pi l(\xi_s'), \quad \int_{D_s} f(\xi_s)\,dv_s' = 4\pi l(\xi_s''). \tag{3}$$

where the primes on ξ_s distinguish the connected-component field ξ_s .

From (2) and (3) we find

$$\int_{O_\varepsilon} \{f(\xi_s)\,dv_s - f(\xi_t)\,dv_t\} = 4\pi\{l(\xi_s') + l(\xi_s'') - l(\xi_t)\}.$$

It is natural to assume that the volume of D_0 is small in comparison with the entire space. We thus have $l(\xi_s') \sim l(\xi_t)$, and $l(\xi_s'')$ agrees in order of magnitude with the linear dimension of the region D_0. Furthermore, for values of t and s sufficiently close to 1/2 in O_ε, we have $dv_s\, dv_t$ 1 by virtue of condition b. By virtue of condition f, however, we then have

$$\int_{O_\varepsilon} f(\xi_s)\,dv_t \gg \int_{O_\varepsilon} f(\xi_s)\frac{dv_s}{dv_t}\,dv_t \sim 4\pi\lambda + \int_{O_\varepsilon} f(\xi_t)\,dv_t.$$

i.e.,

$$\int_{O_\varepsilon} \delta f \cdot dv_t \sim 4\pi\lambda, \tag{4}$$

where $\delta f = f(\xi_s) - f(\xi_t)$.

Introducing the average value of g,

$$< g > = \frac{1}{v_t(O_\varepsilon)} \int_{O_\varepsilon} g\,dv_t,$$

where $v_t(O_\varepsilon)$ is the volume of the region O_ε in the metric $\gamma_{\alpha\beta}(t)$, we can rewrite (4) in the form

$$< \delta f > \cdot v_t(O_\varepsilon) \sim 4\pi\lambda. \tag{5}$$

This relation states that the tearing away of the region D_0 is accompanied by a discontinuity in the curvature of three-space. Since the scalar curvature ${}^{(3)}R$ of three-space can be written [4]

$${}^{(3)}R_t = 2\{K(\xi_t^{\perp}) + 2K(\xi_t)\}.$$

we should assume

$$< \delta^{(3)}R > \sim < \delta f >. \tag{6}$$

From Einstein's equations we have [5]

$$ {}^{(3)}R_t + K_{2,t} = \frac{16\pi G}{c^4}\varepsilon(t), \quad K_{2,t} = (K_\alpha^\alpha)^2 - K_{\alpha\beta}K^{\alpha\beta}, \tag{7} $$

where $K_{\alpha\beta}$ is the external-curvature tensor of the spatial cross section, and $\varepsilon(t)$ is the energy density. By virtue of condition d, the invariant $K_{2,t} = K_{2,t}(x)$, $x \in M_0$, $t \in [0, 1]$ is a continuous function on $M_0 \times [0, 1]$. Consequently, if $\delta K_2 = K_{2,s} - K_{2,t}$, then

$$ < \delta K_2 > = [K_{2,s} - K_{2,t}]|_{x=x_0(t,s)} \xrightarrow[\substack{t\to 1/2-0 \\ s\to 1/2+0}]{} 0. $$

For certain $t_0 < 1/2$ and $1/2 < s_0$, the quantity $< \delta K_2 >$ is therefore negligibly small, and in this case we find from (5)-(7)

$$ < \delta\varepsilon > \sim \frac{c^4}{4\pi G}\frac{\lambda}{v_{t_0}(O_s)}, $$

which may be rewritten as

$$ < \delta\varepsilon > \sim \frac{c^4}{4\pi G}\frac{1}{\sigma}, $$

where σ is a characteristic cross section of the region D_0.

Using this relation we find the following:

1) For $\sigma \sim 10^{20}$ cm² (the sun), we have $< \delta\rho > = < \delta\varepsilon > c^2 \sim 10^7$ g/cm³; 2) for $\sigma \sim 10^{12}$ cm² (a neutron star), we have $< \delta\rho > \sim 10^{15}$ g/cm³; 3) for $\sigma \sim 1$ km², we have $< \delta\rho > \sim 10^{17}$ g/cm³; and 4) for $\sigma \sim 10^{-66}$ cm² (the Big Bang), we have $< \delta\rho > \sim 10^{93}$ g/cm³.

We thus see that the tearing away of small regions is prevented by a high potential barrier. The disruption of the connectedness apparently occurs near singularities of the curvature and near black holes. Neutron stars are nearly in a situation as to be torn away from the surrounding space. These conclusions are in satisfactory agreement with the circumstance that neutron configurations undergo a gravitational collapse when they lose stability.

Comment. Using arguments similar to those above, we could derive conditions for the formation of "handles" on the physical space M. In other words, we could determine the energy expenditure required to disrupt the single-connectedness of the space $(\beta_1(M) = 0 \to \beta_1(M) \neq 0)$.

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