# Relation of uncertainty for time

Alexander K. Guts

Department of Mathematics, Omsk State University
644077 Omsk-77 RUSSIA

E-mail: guts@univer.omsk.su

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ABSTRACT

We introduce two time: deterministic Newton time-stream $t$ and stochastic time-epoch $\tau$. The relation of uncertainty for time-epoch of physical events

$$\Delta\tau\Delta D \geq c_1, \qquad (*)$$

where $c_1 = const$, is proved. The function

$$D(t) = -c_1 \frac{d}{dt} \ln f_\tau(t),$$

characterizes *velocity of disorganization* of the event-phenomena; $f_\tau(t)$ is density of probability of time-epoch $\tau$.

The relation (*) is verified not by means of experiment that is traditional for physics, but with the help of the reference to datas of historical science.

In the World of events $\mathcal{M}$ we select such property as the time order. The time order contacts with such concept as a stream of time. Events are developed (unwrapped) before the observer consistently, in time. It means that for measurement of time the special measuring tool of the *duration* of the phenomenon in time and named a watch is used. With the help of watch to each event the concrete number named (time) moment of event or his(its) epoch is attributed. The time order allows to compare epoch of any events.

However the time stream due to which the phenomena consisting of events are developed (unwrapped) consistently, event behind event, is given to the person as noted philosopher Kant, a priori, from birth. It is consequence of such fact that the person has topologically trivial 4-dimensional body [1, 2]. In other words, time as a stream is only subjective perception (recognition) of the phenomena of the World of the events inherent to the person.

Therefore it is necessary to assume, that time can show itself in our human world, the world of human subjective representations about the World of events, absolutely differently than the time order. As a matter of fact it means, that time can find out itself as something that can *violate* time ordering in deployment of events! Hence events of which the phenomenon consists, can receive epoches with violation of the time order.

Whether means it, what time can have properties similar to a random variable? Anyway it is necessary to try to apply principles of probability theory to the description of time.

We shall accept further that the choice of the epoches (moments) of time which are attributed to events of the phenomenon with the help of some fixed watch can be casual.

Let's forget for simplicity about such concept as a place of event. In this case events in the World of events can be distinguished only with the help of the time order and formally it means that the World of events $\mathcal{M}$ is the linear ordered continuum like real straight line $\mathbb{R}$.

Let's assume that we choose watch $t$ which allow each event $x$ to attribute the moment of time appropriate to him, i.e. epoch $\tau$. We shall accept that each event gets *random epoch*. It is understood as the following. So far as event (atomic event or the elementary phenomenon in the sense of A.D.Alexandrov) is some idealization, it should occupy only an instant $\tau$ in a time-stream $t$. It is accepted in the theory of a relativity. But actually it *is stretched in a time-stream* $t$ and consequently its epoch $\tau$ is absolutely precisely unknown, though must lie on some concrete segment $[\tau, \tau+\Delta\tau]$ of time $t$. Hence epoch $\tau$ of event $x$ is a random variable $\tau :< X, \mathbf{S}, \mathbf{P} > \to \mathbb{R}$, where $X$ is probability space of events, $\mathbf{S}$ is $\sigma$-algebra on $X$, $\mathbf{P}$ is a probability measure on $X$.

Identifying space of events $X$ with the World of events $\mathcal{M}$, and considering that $\mathcal{M}$ is real straight line $\mathbb{R}$, we receive time-epoch $\tau(t)$ as a random variable given in a time-stream $t$.

Event in probability theory is a measurable subset of space $X$. In our terminology the concept of the phenomenon corresponds to concept of event in probability theory. In turn the events which consist of the phenomenon are elements of set $X$ which in probability theory correspond to elementary events. In terminology of Minkowski events are points of the World of events $\mathcal{M}$. But it is obvious that this is simplification accepted in this theory.

So, we shall accept that property of time which is shown in "choice" of the moment of time which corresponds to event is a random variable which we shall name time-epoch.

Let $f_\tau(t)$ be a density of distribution of time-epoch $\tau$ satisfying two conditions

$$\mathbf{M}\tau = \int\limits_{-\infty}^{+\infty} t f_\tau(t)dt = 0 \tag{1}$$

$$\lim_{t\to\pm\infty} t f_\tau(t) = 0 \tag{2}$$

The first condition as it is known is not something important and is connected to a choice of origin of time $t$.

Let

$$D(t) = -c_1 \frac{d}{dt} \ln f_\tau(t), \tag{3}$$

where $c_1 = const$. We have

$$\mathbf{M}D = -c_1 \int\limits_{-\infty}^{+\infty} \left(\frac{d}{dt} \ln f_\tau(t)\right) f_\tau(t)dt = -c_1 \int\limits_{-\infty}^{+\infty} \frac{1}{f_\tau(t)} \frac{df_\tau(t)}{dt} f_\tau(t)dt =$$

$$= -c_1 \int\limits_{-\infty}^{+\infty} df_\tau(t) = -c_1 f_\tau(t)|_{-\infty}^{+\infty} = 0.$$

Therefore a mean square deviation of $D$

$$\Delta D = \sqrt{\mathbf{D}D} = \sqrt{\mathbf{M}D^2 - (\mathbf{M}D)^2} = \sqrt{\mathbf{M}D^2}. \tag{4}$$

Let's find out sense of $D$ defined by the formula (3). As $f_\tau(t)$ density of distribution of size $\tau$ its(her) sense is probability of that event will receive the epoch laying on a piece of a time-stream $[t, t+1]$, where 1 is a standard unit of measurement of time. Let's find out sense of $D$ defined the formula (3). As $f_\tau(t)$ is density of distribution of $\tau$, then its sense is probability of that event will receive the epoch laying on segment $[t, t+1]$ of a time-stream, where 1 is a standard unit of measurement of time.

But then by analogy to the Boltzmann formula for entropy, it is possible to declare that $-c_1 \ln f_\tau(t)$ is entropy of time-epoch. In other words, it characterizes a measure of disorganization of event as the phenomenon. Therefore $D(t)$ characterizes *velocity of disorganization* the event-phenomenon.

As it will be shown below this velocity the is more, than temporal borders are closer for localization of the phenomenon in a stream of time. We can deduce now some law to which time-epoch satisfies.

**Theorem.** *If the conditions* (1), (2) *are held, then relation of uncertainty*

$$\Delta\tau\Delta D \geq c_1 \tag{5}$$

*is true.*

**Proof.**

To prove (5) we apply the method of Weyl [3, p.69-70].

We have the inequality

$$0 \leq \int_{-\infty}^{+\infty} \left(\alpha t\sqrt{f_\tau(t)} + \frac{d}{dt}\sqrt{f_\tau(t)}\right)^2 dt =$$

$$= \alpha^2 \int_{-\infty}^{+\infty} t^2 f_\tau(t)dt + 2\alpha \int_{-\infty}^{+\infty} t\sqrt{f_\tau(t)}\frac{d}{dt}\sqrt{f_\tau(t)}dt + \int_{-\infty}^{+\infty} \left(\frac{d}{dt}\sqrt{f_\tau(t)}\right)^2 dt \tag{6}$$

Let's calculate each of integrals in the right part of inequality (6). First of all, it follows from (1)

$$\int_{-\infty}^{+\infty} t^2 f_\tau(t)dt = \mathbf{M}\tau^{\mathbf{2}} = \mathbf{M}\tau^{\mathbf{2}} - (\mathbf{M}\tau)^2 = \mathbf{D}\tau. \tag{7}$$

By using (2.2) we get

$$2\int_{-\infty}^{+\infty} t\sqrt{f_\tau(t)}\frac{d\sqrt{f_\tau(t)}}{dt}dt = \int_{-\infty}^{+\infty} t\frac{d(\sqrt{f_\tau(t)}\sqrt{f_\tau(t)})}{dt}dt = \int_{-\infty}^{+\infty} tdf_\tau(t) =$$

$$= tf_\tau(t)|_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty} f_\tau(t)dt = -1. \tag{8}$$

And, at last, we have for the third integral

$$\int_{-\infty}^{+\infty} \left(\frac{d}{dt}\sqrt{f_\tau(t)}\right)^2 dt = \int_{-\infty}^{+\infty} \left(\frac{1}{\sqrt{f_\tau(t)}}\frac{d\sqrt{f_\tau(t)}}{dt}\right)^2 f_\tau(t)dt =$$

$$= \int_{-\infty}^{+\infty} \left(\frac{d}{dt}\ln\sqrt{f_\tau(t)}\right)^2 f_\tau(t)dt = \frac{1}{4c_1^2}\int_{-\infty}^{+\infty}\left(c_1\frac{d}{dt}\ln f_\tau(t)\right)^2 f_\tau(t)dt =$$

$$= \frac{1}{4c_1^2}\mathbf{M}D^2 = \frac{1}{4c_1^2}(\Delta D)^2. \tag{9}$$

Thus from (6)-(9) we have an inequality

$$\alpha^2(\Delta\tau)^2 - 1 + \frac{1}{4c_1^2}(\Delta D)^2 \geq 0,$$

which must be true for all $\alpha$. Hence

$$1 - 4(\Delta\tau)^2\frac{1}{4c_1^2}(\Delta D)^2 \leq 0$$

or

$$\Delta\tau\Delta D \geq c_1.$$

Theorem is proved.

The relation of uncertainty (5) was postulated in [5, 6, 7] as one of laws of time. The name was given to it: *the law of uncertainty of descriptions*. It was formulated on the basis of the analysis of the historical sources used by historians for descriptions of events of the past. Thus the relation (5) is proved not by means of experiment thai is traditional for physics, but with the help of the reference to datas of historical science.

Though without any doubt after formalization of concepts $\Delta\tau, \Delta D$ that it was not made in [5, 6, 7] it is possible to speak and about experimental check of relation (5).

Basis for reception of our result was that circumstance that space-time $\mathcal{M}$ has a dual nature which was incorporated by the founder of this theory Minkowski [4]. This duality consists of that on the one hand elements of set $\mathcal{M}$ are (atomic) *events*, and by virtue of it $\mathcal{M}$ carries the name the World of events, and on the other hand $\mathcal{M}$ is *arithmetic* arena on which the World of events is realized.

This arithmetic arena is necessary for formalization of the World of events to attribute to events the coordinates as the four of real numbers, to world lines of the four of real functions etc. As a rule the researcher deals with mathematical space-time which we have named arithmetic arena.

However other side of space-time, the World of events, remained in a shadow and was not formalized! At the beginning of article we have identified $\mathcal{M}$ with probability space of events $X$. Given probability space $X$ is the formalized World of events.

In other words it is necessary to look at space-time $\mathcal{M}$ from two sides, on the one hand as on set of the elements named elementary (atomic) events and to say that *event* is any (measurable) subset of set $\mathcal{M}$ as it is accepted in probability theory; on the other hand as on coordinate space (arithmetic arena), for example, $\mathbb{R}^4$ which is used for formalization of concepts of the World of events.

Let's notice, that at the description of deterministic processes and the phenomena, and also at the description of stochastic processes and the phenomena which are not touching a nature as of space, and time, we use only the second point of view on the World of events, but the stochastic processes concerning of nature of time require to distinguish two sides of the World of events.

Above we used space-time as coordinate space that elementary event could receive epoch on "an axis of time". This epoch does not lie in "strictly allocated place" according to the "instruction" of the time order, but can occupy any place on "an axis of time" not especially caring of instructions of the mentioned time order. This concerns and to spatial coordinates of events. So with the formula (5) it is possible to deduce similar formulas for mean square deviations of $x-, y-$ and $z-$ coordinates of events.

Let's note one more circumstance. Time as it is found out in this work can be not only *deterministic time-stream* connected with classical representation of Newton about time as about duration and, accordingly, with concept of the time order, but can be *stochastic time-epoch*, having such characteristic as *density* probability.

The last sets in the certain sense intensity of display (demonstration) of events of the phenomenon on a segment of uniformly (current) time-stream. Here it would be pertinent to recollect that N.A.Kozyrev wrote about *density of time* describing its intensity in his articles [8]. And though in our case the question is stochastic properties of time nevertheless it is possible to be surprised the intuition of Pulkov astronomer.

## References


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