THE RELATION OF UNCERTAINTY FOR RADIUS OF THE UNIVERSE

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We get the relation of uncertainty \( \Delta R \Delta D \sim c_2 \) connecting the mean square derivations \( \Delta R \) and \( \Delta D \) of radius of the Universe and velocity disorganization of event as the phenomena respectively.

In the World of events \( \mathcal{M} \) we select such property as the time order. The time order contacts with such concept as a stream of time. Events are developed (unwrapped) before the observer consistently, in time. It means that for measurement of time the special measuring tool of the duration of the phenomenon in time and named a watch is used. With the help of watch to each event the concrete number named (time) moment of event or his epoch is attributed. The time order allows to compare epochs of any events.

However the time stream due to which the phenomena consisting of events are developed (unwrapped) consistently, event behind event, is given to the person as noted philosopher Kant, a priori, from birth. In other words, time as a stream is only subjective perception (recognition) of the phenomena of the World of the events inherent to the person.

Therefore it is necessary to assume, that time can show itself in our human world, the world of human subjective representations about the World of events, absolutely differently than the time order. As a matter of fact it means, that time can find out itself as something that can violate time ordering in deployment of events! Hence events of which the phenomenon consists, can receive epochs with violation of the time order.

Whether means it, what time can have properties similar to a random variable? Anyway it is necessary to try to apply principles of probability theory to the description of time.

We shall accept further that the choice of the epochs (moments) of time which are attributed to events of the phenomenon with the help of some fixed watch can be casual.

Let’s forget for simplicity about such concept as a place of event. In this case events in the World of events can be distinguished only with the help of the time order and formally it means that the World of events \( \mathcal{M} \) is the linear ordered continuum like real straight line \( \mathbb{R} \).

Let’s assume that we choose watch \( t \) which allow each event \( x \) to attribute the moment of time appropriate to him, i.e. epoch \( \tau \). We shall accept that each event gets random epoch. It is understood as the following. So far as event is some idealization, it should occupy only an instant \( \tau \) in a time-stream \( t \). It is accepted in the theory of a relativity. But actually it is stretched in a time-stream \( t \) and consequently its epoch \( \tau \) is absolutely precisely unknown, though must lie on some concrete segment \( [\tau, \tau + \Delta \tau] \) of time \( t \). Hence epoch \( \tau \) of event \( x \) is a random variable \( \tau : \mathcal{M} \rightarrow \mathbb{R} \), where \( X \) is probability space of events, \( \mathcal{S} \) is \( \sigma \)-algebra on \( X \), \( \mathbb{P} \) is a probability measure on \( X \).

Identifying space of events \( X \) with the World of events \( \mathcal{M} \), and considering that \( \mathcal{M} \) is real straight line \( \mathbb{R} \), we receive time-epoch \( \tau(t) \) as a random variable given in a time-stream \( t \).

Event in probability theory is a measurable subset of space \( X \). In our terminology the concept of the phenomenon corresponds to concept of event in probability theory. In turn the events which consist of the phenomenon are elements of set \( X \) which in probability theory correspond to elementary events. In terminology of Minkowski events are points of the World of events \( \mathcal{M} \). But it is obvious that this is simplification accepted in this theory.

So, we shall accept that property of time which is shown in "choice" of the moment of time which corresponds to event is a random variable which we shall name time-epoch.

Let \( f_\tau(t) \) be a density of distribution of time-epoch \( \tau \) satisfying two conditions

\[
\text{M}_\tau = \int_{-\infty}^{+\infty} f_\tau(t) dt = 0; \quad \lim_{t \to +\infty} t f_\tau(t) = 0. \tag{1}
\]

Let

\[
D(t) = -c_1 \frac{d}{dt} \ln f_\tau(t), \tag{2}
\]
where \( c_1 = \text{const} \).

Let's find out sense of \( D \) defined the formula (2). As \( f_\tau(t) \) is density of distribution of \( \tau \), then its sense is probability of that event will receive the epoch laying on segment \([t, t+1] \) of a time-stream, where 1 is a standard unit of measurement of time.

But then by analogy to the Boltzmann formula for entropy, it is possible to declare that \(-c_1 \ln f_\tau(t)\) is entropy of time-epoch. In other words, it characterizes a measure of disorganization of event as the phenomenon. Therefore \( D(t) \) characterizes velocity of disorganization the event-phenomenon.

As it will be shown below this velocity the is more, than temporal borders are closer for localization of the phenomenon in a stream of time.

The following theorem is valid.

**Theorem.** If the conditions (1) are held, then relation of uncertainty

\[
\Delta \tau \Delta D \geq c_1
\]

is true.

In cosmology a special role one play the Friedman's solutions of the form

\[
ds^2 = dt^2 - R^2(t_0 + t)[d\chi^2 + S^2(\chi)d\Omega^2],
\]

\[-t_0 < t < +\infty,
\]

where we changed the temporal origin of the Universe: \( 0 \to -t_0 \). Here the time parameter \( t \) is directly connected to postulation of evolution of the universe, and, hence, any varied property of the universe plays a role of hours. The \( t = 0 \) is moment of measurement of the Universe radius.

If to admit that the radius \( R = R(\tau) \) of the universe is a random variable which is similar to time-epoch \( \tau \), and which can be calculated by means of known methods, then mean square derivation \( \Delta R \) will receive through \( \Delta \tau \).

For the critical dust Friedman model \( R(t_0 + t) = A + \beta t + \delta(t) \). Hence in the first approximation \( \Delta R = \Delta \tau \) and

\[\Delta R\Delta D \sim c_2.\]

So value of measured radius of the Universe will depend on such characteristics of the universal phenomena (events) as velocity of their becoming or destruction.

Can the moment \( t \) of measurement of the Universe radius be arbitrary? No, the law (2) is valid only for the past epochs [1, 2, 3, 4].

Basis for reception of our result was that circumstance that space-time \( \mathcal{M} \) has a dual nature which was incorporated by the founder of this theory Minkowski [5]. This duality consists of that on the one hand elements of set \( \mathcal{M} \) are (atomic) events, and by virtue of it \( \mathcal{M} \) carries the name the World of events, and on the other hand \( \mathcal{M} \) is arithmetic arena on which the World of events is realized.

This arithmetic arena is necessary for formalization of the World of events to attribute to events the coordinates as the four of real numbers, to world lines of the four of real functions etc. As a rule the researcher deals with mathematical space-time which we have named arithmetic arena.

However other side of space-time, the World of events, remained in a shadow and was not formalized! At the beginning of article we have identified \( \mathcal{M} \) with probability space of events \( X \). Given probability space \( X \) is the formalized World of events.

Above we used space-time as coordinate space that elementary event could receive epoch on "an axis of time". This epoch does not lie in "strictly allocated place" according to the "instruction" of the time order, but can occupy any place on "an axis of time" not especially caring of instructions of the mentioned time order.

Let's note one more circumstance. Time as it is found out in this work can be not only deterministic time-stream connected with classical representation of Newton about time as about duration and, accordingly, with concept of the time order, but can be stochastic time-epoch, having such characteristic as density probability.

The last sets in the certain sense intensity of display (demonstration) of events of the phenomenon on a segment of uniformly (current) time-stream. Here it would be pertinent to recollect that N.A. Kozyrev wrote about density of time describing its intensity in his articles [6]. And though in our case the question is stochastic properties of time nevertheless it is possible to be surprised the intuition of Pulkov astronomer.

**References**


