

Time machine as four-dimensional wormhole

Alexandr K. Guts

Department of Mathematics, Omsk State University
644077 Omsk-77 RUSSIA

E-mail: guts@univer.omsk.su

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ABSTRACT

The following mechanism of action of Time machine is considered. Let space-time $\langle V^4, g_{ik} \rangle$ be a leaf of a foliation \mathcal{F} of codimension 1 in 5-dimensional Lorentz manifold $\langle V^5, G_{AB} \rangle$. If the Godbillon-Vey class $GV(\mathcal{F}) \neq 0$ then the foliation \mathcal{F} has resilient leaves. Let V^4 be a resilient leaf. Hence there exists an arbitrarily small neighborhood $U_a \subset V^5$ of the event $a \in V^4$ such that $U_a \cap V^4$ consists of at least two connected components U_a^1 and U_a^2 .

Remove the four-dimensional balls $B_a \subset U_a^1, B_b \subset U_a^2$, where an event $b \in U_a^2$, and join the boundaries of formed two holes by means of 4-dimensional cylinder. As result we have a four-dimensional wormhole C , which is a Time machine if b belongs to the past of event a . The past of a is lying arbitrarily nearby. The distant Past is more accessible than the near Past. It seems that real global space-time V^4 is a resilient one, i.e. is a resilient leaf of some foliation \mathcal{F} .

It follows from the conformal Kaluza-Klein theory that the movement to the Past through four-dimensional wormhole C along geodesic with respect to metric G_{AB} requires for time machine of large energy and electric charge.

We have a Time machine in a space-time domain D , when a smooth closed time-like curve exists in this domain. In the papers [1, 2] the case of creation of Time machine from the three-dimensional wormhole (3-wormhole) by means of kinematic procedures with one mouth of wormhole is considered. The author agrees with opinion of M. Ju. Konstantinov [3, 4] that this assertion is erroneously because contradict to Principle of equivalence and "in according with theorems about global hyperbolicity and Cauchy problem the models with causality violation could not be considered as the result of dynamical evolution of some initial space-like configuration and must be considered as a solution of some boundary problem" [4]. But Time machine exists if solution of the Einstein's equations with 3-wormhole contains a closed time-like curve, i.e. if under creation of 3-wormhole simultaneously the closed time-like curve is created.

In this paper the different principle of action of the Time machine is discussed. The closed time-like curve can be constructed by means of attaching of four-dimensional handle (4-wormhole).

1 Conditions of existence of Time machine in space-time with Euclidean topology

Let g_{ik} ($i, k = 0, \dots, 3$) be a constant gravitational field in domain D , which is homeomorphic to Euclidean space \mathbb{R}^4 and let this field is created by dust. Suppose that $g_{00} > 0$ and what is more $g_{00} = const = 1$. The last relation can be reached by means of transformation

$$\bar{x}^0 = \int \sqrt{g_{00}} dx^0, \quad \bar{x}^\alpha = x^\alpha, \quad (\alpha = 1, 2, 3).$$

Assume that closed time-like curve L is analytic Jordan curve and it lies on 2-dimensional oriented surface $F \subset D$. And what is more suppose that L is border of F .

It follows from formula [5, p.357] that

$$g_{\alpha\beta} f_{\alpha\beta} f^{\alpha\beta} = \frac{16\pi G\rho}{c^2} \frac{1 + \frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}}, \quad (1.1)$$

where

$$f_{\alpha\beta} = \frac{\partial}{\partial x^\alpha} \left(\frac{g_{0\beta}}{g_{00}} \right) - \frac{\partial}{\partial x^\beta} \left(\frac{g_{0\alpha}}{g_{00}} \right),$$

ρ is density of dust, v is velocity of dust.

The formula (1.1) can be rewrite in the form

$$\begin{aligned} & \Delta_3^3(f_{12})^2 + \Delta_2^2(f_{13})^2 + \Delta_1^1(f_{23})^2 + 2\Delta_2^3f_{12}f_{13} + 2\Delta_1^3f_{12}f_{23} + \\ & + 2\Delta_1^2f_{13}f_{23} = \frac{16\pi G\rho}{g_{00}c^2} \frac{1 + \frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}}, \end{aligned} \quad (1.2)$$

where Δ_β^α is minor of 2nd order of matrix $\| -g^{\alpha\beta} \|$ that is got by means of crossing out of line with number α and column with number β .

Further we consider only such gravitational fields for which either all

$$\Delta_\beta^\alpha \geq 0, \quad (1.3)$$

or, if $\Delta_\beta^\alpha < 0$ then in (1.2) the summand containing this Δ_β^α is equal to zero. The condition (1.3) is true for the Gödel's solution, and the second, for example, for metric

$$\begin{aligned} ds^2 = & \frac{1}{2}(dx^0)^2 + 2\Omega x^2 dx^0 dx^1 - \Omega x^1 dx^0 dx^2 + \left(\Omega^2 (x^2)^2 - \frac{1}{2} \right) (dx^1)^2 - \\ & - 2\Omega^2 x^1 x^2 dx^1 dx^2 + \left(\Omega^2 (x^1)^2 - \frac{1}{2} \right) (dx^2)^2 + A(dx^3)^2, \end{aligned}$$

which contains smooth closed time-like curve of the form $x^1 = a \sin \mu$, $x^2 = a \cos \mu$, $x^0, x^3 = const$.

If $f_{\alpha\beta} < 0$ ($\alpha < \beta$) then we make the change $f_{\alpha\beta} = -f_{\beta\alpha}$ and get that all $f_{\alpha\beta}$ in (1.2) will be non-negative.

Let

$$\Delta = \min_{\alpha, \beta} \inf_D \Delta_\beta^\alpha > 0,$$

where we considered only non-zero minors. It follows from (1.2) that

$$(f_{12} + f_{13} + f_{23})^2 \Delta \leq \frac{16\pi G\rho}{g_{00}c^2} \frac{1 + \frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}}. \quad (1.4)$$

Now we are able to calculate the chronometrical invariant time $t(L)$ of passage of curve L [6, 7]. In the constant gravitational field $f_{0i} = 0$. By using this equality and the Stokes's formula me get

$$t(L) = \frac{1}{c} \int_L \frac{g_{0i} dx^i}{\sqrt{g_{00}}} =$$

$$\begin{aligned}
&= \frac{\sqrt{g_{00}}}{c} \iint_F f_{12} dx^1 dx^2 + f_{13} dx^1 dx^3 + f_{23} dx^2 dx^3 = \\
&= \frac{\sqrt{g_{00}}}{c} \iint_F (f_{12} \cos \theta_1 + f_{13} \cos \theta_2 + f_{23} \cos \theta_3) dS.
\end{aligned}$$

By using (1.4) we have further

$$\begin{aligned}
t(L) &\leq \frac{\sqrt{g_{00}}}{c} \iint_F (f_{12} + f_{13} + f_{23}) dS \leq \\
&\leq \frac{4\sqrt{\pi G}}{c^2} \iint_F \left(\frac{\rho}{\Delta} \frac{1 + \frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}} \right)^{\frac{1}{2}} dS.
\end{aligned}$$

Suppose that density ρ and velocity v of dust are constant in domain D . Then

$$t(L) \leq \frac{4}{c^2} \left(\frac{\pi G \rho}{\Delta} \frac{1 + \frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}} \right)^{\frac{1}{2}} \sigma(F), \quad (1.5)$$

where

$$\sigma(F) = \iint_F dS$$

is "Euclidean" area of surface F .

If $s(L)$ is proper time of L then

$$s(L) = \int_L \left(1 - \frac{V^2}{c^2} \right)^{\frac{1}{2}} dt,$$

where V is velocity of Time machine with world line L ,

$$V^2 = \gamma_{\alpha\beta} V^\alpha V^\beta, \quad V^\alpha = \frac{dx^\alpha}{dt},$$

$$\gamma_{\alpha\beta} = -g_{\alpha\beta} + \frac{g_{0\alpha} g_{0\beta}}{g_{00}}.$$

Hence proper time of return to the past can be less than any positive small number. But then the velocity of Time machine must tend to the velocity of light.

The formula (1.5) generalizes (Demidov V.V. Master's degree work, Omsk State University, 1986. – see [6, 7]) the analogous estimate

$$t(L) \sim \frac{\sqrt{8\pi G\rho}}{c^2} \sigma(F) \quad (1.6)$$

from [8] which was got under the assumptions: $g_{03} = g_{13} = g_{23} = 0$, the time loop lies on the "plane" $F = \{x^0, x^3 = const\}$ and $v = 0$. It follows from (1.6) that if it is true "Euclidean" relation

$$\sigma(F) \sim \pi^{-1} [l(L)]^2, \quad (1.7)$$

where

$$l(L) = \int_L \sqrt{\gamma_{\alpha\beta} dx^\alpha dx^\beta}$$

is 3-length of L and $\sigma(F)$ is area of F , then

$$t(L) \sim 2 \cdot 10^{-24} \sqrt{\rho} \cdot [l(L)]^2 \text{ (sec)}.$$

Hence under $\rho \sim 10^{-31} g/cm^3$ and $t \sim 1 \text{ year}$ we have $l \sim [\text{distance between Sun and Galaxy's center}] \sim 8000 \text{ parsec}$; if $l \sim 1000 \text{ km}$, then $t \sim 6 \cdot 10^{-23} \text{ sec}$! When $t \sim 1 \text{ year}$ and $l \sim 1000 \text{ km}$, then $\rho \sim 6 \cdot 10^{28} g/cm^3$!!

If we throw off the relation (1.7) then under $t \sim 1 \text{ year}$, $l \sim 1000 \text{ km}$ and $\rho \sim 10^{-31} g/cm^3$ we get $\sigma \sim 10^9 \pi^{-1} l^2$, i.e. deviation from Euclidean geometry in space where exists smooth closed time-like curve is very vast. This means that Time machine is realized in such domains where act strong gravitational fields which destroy the human organism.

Suppose that functions $g_{ik} \in C^1(D)$. It is easy to prove [7] that in considered domain D the existence of smooth closed time-like curves which are homotopic to zero and are passing through given point $x_0 \in D$ implies the equality

$$\det || g_{ik}(x_0) || = 0,$$

i.e. we have a singularity. Hence the smooth closed time-like curves which are homotopic to zero (or are contractible to point) do not exist in non-singular space-time with Euclidean topology.

2 The process of creation of wormholes in space-time

The closed time-like curve can be constructed by means of the topological change of domain D or more exactly by means of attaching of 4-dimensional handle (4-wormhole).

How does it get the change of topology in the given compact domain D of space-time? The answer is contained in the consideration of some analogy of the Gauss-Bonnet's formula for closed 3-dimensional space-like surface $V^3 \subset D$ (the closure can be got by identification the points of border $\partial V^3 \subset \partial D$ of non-closed 3-surface V^3):

$$\iiint_{V^3} K(g_{\alpha\beta}, \frac{\partial g_{\alpha\beta}}{\partial x^\nu}, \frac{\partial^2 g_{\alpha\beta}}{\partial x^\nu \partial x^\mu}) dv = \sum_{\nu=0}^3 c_\nu b_\nu(t), \quad (2.1)$$

where K is the some function of 3-metric $g_{\alpha\beta}$ and its derivatives, t is the time parameter, b_ν is the ν -dimensional Betti number, c_ν is the real costant [9, 10, 11].

The change of connectivity or simple-connectivity of 3-surface V^3 at the moment $t = t_0$ is realized if $b_0(t)$ or $b_1(t)$ are changed by jump by changing $t, t_0 - \delta < t < t_0 + \delta, \delta > 0$. It doesn't take place if the curve $L : t \rightarrow g_{\alpha\beta}(t)$ which belongs to the space of 3-metrics equipped with $C^2(D)$ -topology (it means the closeness with respect to the first and second derivatives of $g_{\alpha\beta}$) is continuous. The discontinuity of the curve L means the existence of discontinuities for the second derivatives of $g_{\alpha\beta}$ on the set $A \subset V^3$. So the topology of V^3 is changed if 3-metric $g_{\alpha\beta}(t) = g_{\alpha\beta}(t, x^1, x^2, x^3)$ undergoes the discontinuities of the second derivatives under $t = t_0$.

Since $g_{\alpha\beta}(t)$ must satisfy the Einstein's equations then the discontinuities of $\partial_k \partial_l g_{\alpha\beta}$ take the place when the certain energy sources with continuous energy-momentum tensor are switched on at the moment $t = t_0$ [9]. The discontinuities of $\partial_k \partial_l g_{\alpha\beta}$ are the consequence either shock gravitational waves or another waves the velocity of propagation of which is less than the velocity of light.

In detail this construction of 4-wormhole creation was analyzed in papers [10, 11]. For example [10], the formula (2.1) for closed orientable Riemannian manifold V^3 with Killing unit vector field ξ has the form [12]

$$\frac{1}{2\pi l(\xi)} \iiint_{V^3} [K(\xi_\perp) + 3K(\xi)] dv = 2b_0(V^3) - b_1(V^3) + d_0,$$

where $K(\xi_\perp)$ is Riemannian curvature in the plane that is orthogonal to ξ , $K(\xi)$ is Riemannian curvature in the arbitrary plane containing ξ , $l(\xi)$ is the length integral trajectory of field ξ , $d_0 = 0$ if b_1 is even and $d_0 = 1$ if b_1 is odd. In the paper [11] the case of non-compact 3-space without any symmetries is considered.

In the case of the General Theory of Relativity the following general result takes the place. If σ is the characteristic 2-dimensional section of the 3-dimensional domain D_0 that contains the 4-wormhole than the mean value of energy density jump [10, 11]

$$\langle \delta\epsilon \rangle \sim \frac{c^4}{4\pi G} \frac{1}{\sigma},$$

where c is the light velocity, G is the gravitational constant.

The creation of 4-wormhole means that 3-dimensional piece D_0 leaves the 3-dimensional physical space V^3 or is separated from V^3 .

3 Resilient space-time

When we have the chance to create the 4-wormhole going from the Present to the Past? It is evidently if the temporal stream carries out the peace D_0 to the Past which is lying arbitrarily nearby. We can realize this by means of the theory of resilient leaves of foliations of codimension 1 in the 5-dimensional Lorentz manifold and the conformal Kaluza-Klein theory [13, 15].

Let $\langle V^4, g_{ik} \rangle$ be a leaf of an orientable foliation \mathcal{F} of codimension 1 in the 5-dimensional Lorentz manifold $\langle V^5, G_{AB} \rangle$, $g = G|_{V^4}$, $A, B = 0, 1, 2, 3, 5$. Foliation \mathcal{F} is determined by the differential 1-form $\gamma = \gamma_A dx^A$. If the Godbillon-Vey class $GV(\mathcal{F}) \neq 0$ then the foliation \mathcal{F} has a resilient leaves [16].

We suppose that real global space-time V^4 is a resilient one, i.e. is a resilient leaf of some foliation \mathcal{F} . Hence there exists an arbitrarily small neighborhood $U_a \subset V^5$ of the event $a \in V^4$ such that $U_a \cap V^4$ consists of at least two connected components U_a^1 and U_a^2 .

Remove the 4-dimensional balls $B_a \subset U_a^1, B_b \subset U_a^2$, where an event $b \in U_a^2$, and join the boundaries of formed two holes by means of 4-dimensional cylinder. As result we have a 4-wormhole C , which is a Time machine if b belongs to the past of event a . The past of a is lying arbitrarily nearby. The distant Past is more accessible than the near Past. A movement along 5-th coordinate (in the direction γ^A) gives the infinite piercing of space-time V^4 at the points of Past and Future. It is the property of a resilient leaf [16].

Define the Lorentz metric \tilde{G}_{AB} on V^5 in the following way

$$\tilde{G}_{AB} = -\gamma_A\gamma_B + \tilde{g}_{AB},$$

$$\tilde{g}_{5A} = 0,$$

where \tilde{g}_{AB} is metric tensor of V^4 . It is more convenient [17] to use conformal metric G_{AB}

$$G_{AB} = \phi^{-2}\tilde{G}_{AB}, \quad g_{AB} = \phi^{-2}\tilde{g}_{AB}, \quad \phi = \gamma_5,$$

$$G_{AB} = -\lambda_A\lambda_B + g_{AB},$$

$$\lambda = \phi^{-1}\gamma,$$

$$(d\tilde{I})^2 = \tilde{G}_{AB}dx^A dx^B = \phi^2 G_{AB}dx^A dx^B = \phi^2 dI^2$$

with the cylindrical condition that G_{AB} are not depend of x^5 and $G_{55} = -1$. Than ϕ is a scalar field, and the 5-dimensional Einstein's equations

$$R_{AB}^{(5)} - \frac{1}{2}G_{AB}R^{(5)} = \kappa Q_{AB}$$

are reduced [17] to the 4-dimensional Einstein's equations, the Maxwell's equations and the Klein-Fock equation for ϕ

$$\begin{aligned} R_{ik}^{(4)} - \frac{1}{2}g_{ik}R^{(4)} - \Lambda\phi^2 g_{ik} = & -\frac{2G}{c^4}(F_{im}F_{.k}^{.m} - \frac{1}{4}g_{ik}F_{mn}F^{mn}) + \\ & + \frac{3}{\phi}(\nabla_i\nabla_k\phi - g_{ik}g^{mn}\nabla_m\nabla_n\phi) - \frac{6}{\phi^2}\phi_{,i}\phi_{,k} + \kappa Q_{ik}, \end{aligned} \quad (3.1)$$

$$-\nabla_m F^{mk} - 3F^{mk}\frac{\phi_{,m}}{\phi} = \frac{c^2\kappa}{\sqrt{G}}\phi^3 Q_A^k \lambda^A,$$

$$g^{mn}\nabla_m\nabla_n\phi - \frac{1}{6}R^{(4)}\phi + \frac{1}{3}\Lambda\phi^3 - \frac{G\phi}{2c^4}F_{mn}F^{mn} = -\frac{\kappa}{3}\phi^3 Q_{AB}\lambda^A\lambda^B$$

where $\kappa = 8\pi G/c^4$, $\phi_{,i} = \partial\phi/\partial x^i$. The co-vector γ_A is determinated by the scalar field ϕ and 4-potential λ_i , $i = 0, 1, 2, 3$ of electro-magnetic field $F_{ik} = \lambda_{i,k} - \lambda_{k,i}$.

Define the proper time along time-like curve L as

$$d\tau = \frac{dI}{c}, \quad dI^2 = G_{AB}dx^A dx^B,$$

where coordinates $x^A (A = 0, 1, 2, 3, 5)$ are given in domain $U_a \subset V^5$. Suppose that Time machine moves in the Past along L in U_a such that it is at rest in the domain $V^4 \cap U_a$, i.e. $x^1, x^2, x^3 = \text{const}$. Then

$$dI^2 = ds^2 - d\lambda^2, \quad ds^2 = c^2 dt^2 - dl^2,$$

where

$$dt = \frac{g_{0i} dx^i}{c\sqrt{g_{00}}}, \quad dl^2 = \left(-g_{\alpha\beta} + \frac{g_{0\alpha}g_{0\beta}}{g_{00}} \right) dx^\alpha dx^\beta \quad (\alpha, \beta = 1, 2, 3)$$

are chronometrical invariant respectively time and length in space-time V^4 . Hence

$$d\tau = \sqrt{1 - \left(\frac{d\lambda}{ds} \right)^2} \frac{ds}{c} = \sqrt{1 - \left(\frac{d\lambda}{ds} \right)^2} \sqrt{1 - \left(\frac{dl}{cdt} \right)^2} dt = \sqrt{1 - \left(\frac{d\lambda}{ds} \right)^2} dt, \quad (3.2)$$

since $dl = 0$.

Let L be a time-like geodesic curve with respect to 5-metric G_{AB} that has the ends a and b .

Then [17, c.51]

$$\frac{d\lambda}{ds} = -\frac{e}{2m\sqrt{G}}, \quad (3.3)$$

where e is electric charge, and m is mass of Time machine.

If any vector ξ is tangent to V^4 then $d\lambda(\xi) = \lambda_A \xi^A = 0$. Hence the motion along curve $L : x^A = x^A(s)$ and which is transversal to W^4 is characterized by means of the inequality

$$d\lambda\left(\frac{dx^A}{ds}\right) = \lambda_A \frac{dx^A}{ds} = \frac{d\lambda}{ds} \neq 0. \quad (3.4)$$

The relations (3.3) and (3.4) imply that for transversal motion, i.e. motion in 5-th dimension it is necessary that body had an electric charge. Therefore for start of Time machine we must give to it the electric charge.

It follows from (3.2) that $(d\lambda/ds)^2 \leq 1$, because time τ must be real. Hence $e/2m\sqrt{G} \leq 1$. This inequality is not correct for electron. Thus there exists the restriction for mass and electric charge of Time machine.

In the case when the Godbillon-Vey class $GV(\mathcal{F}) = 0$ one can attempt to change this, i.e. to include the foliation \mathcal{F} in smooth one-parametric family of foliations \mathcal{F}_μ characteristic class

$$\text{Char}_{\mathcal{F}_\mu}(\alpha) = GV(\mathcal{F}_\mu), \alpha \in H^3(W_1),$$

$$Char_{\mathcal{F}_\mu}(\alpha) : H^*(W_1) \rightarrow H^*(V^5, \mathbb{R})$$

of which is changed with change of parameter μ in accord to law

$$GV(\mathcal{F}_\mu) \frac{d}{d\mu} GV(\mathcal{F}_\mu) = 0.$$

Change (variation) is possible if $\alpha \in H^3(W_1)$ does not belong to image of homomorphism of inclusion $H^3(W_2) \rightarrow H^3(W_1)$ [19]. Since $H^3(W_2) = 0$ [20] then one can be taken arbitrary cohomological class $\alpha \neq 0$.

Suppose that our space-time is not resilient one. Can it be rolled up? It is very difficult question. As we think it has positive answer. This is consequence of the Antropological Principle.

4 Source of energy for Time machine

It follows from 00-equation (3.1) that

$$R^{(3)} + K_2 = \frac{4G}{c^4} \varepsilon_F(t) + 2\varepsilon_\phi(t) + \frac{16\pi G}{c^4} \varepsilon_Q(t). \quad (4.1)$$

where $R^{(3)}$ is scalar curvature and K_2 is exterior one (with respect to V^4) of 3-space V^3 , $\varepsilon_F(t)$, $\varepsilon_\phi(t)$, $\varepsilon_Q(t)$ are energy densities respectively of electro-magnetic field, scalar field and the other matter. Hence the jumps of energy densities are consequence of the jump of curvature $\langle \delta R^{(3)} \rangle \sim 2$ (we suppose that $\langle \delta K_2 \rangle = 0$), which defines the change of topology of 3-space (see §1 and [10, 11]) and separation of domain D_0 with the characteristic 2-dimensional section σ from space V^3 . The relation (4.1) shows that the jumps of energy of electro-magnetic field and the other matter can be very vast

$$\langle \delta \varepsilon_F \rangle \sim \frac{c^4}{G} \frac{1}{\sigma}, \quad \langle \delta \varepsilon_Q \rangle \sim \frac{c^4}{4\pi G} \frac{1}{\sigma},$$

but for scalar field $\langle \delta \varepsilon_\phi \rangle \sim 1$.

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